Dipolar quantum gases

Danny Baillie

Physics, University of Otago

10 February 2016

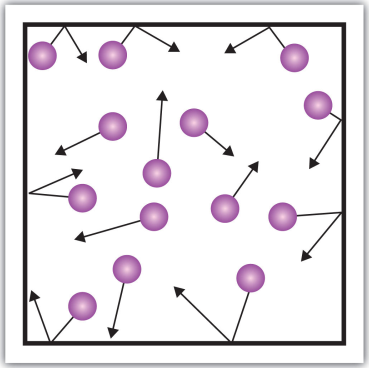
The Dodd-Walls Centre
for Quantum Science and Technology

MARSDEN FUND
TE PŪTEA RANGAHAU A MARSDEN

UNIVERSITY of OTAGO
SAPERE AUDE
Te Whare Wānanga o Otāgo
NEW ZEALAND

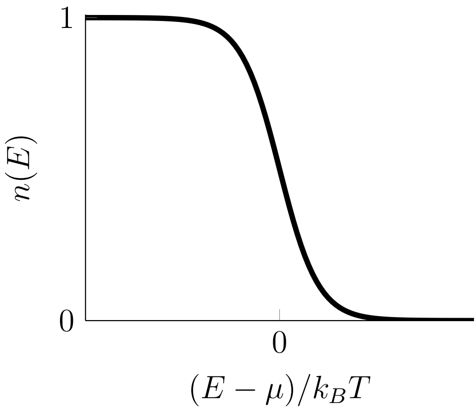
Gases

Gases



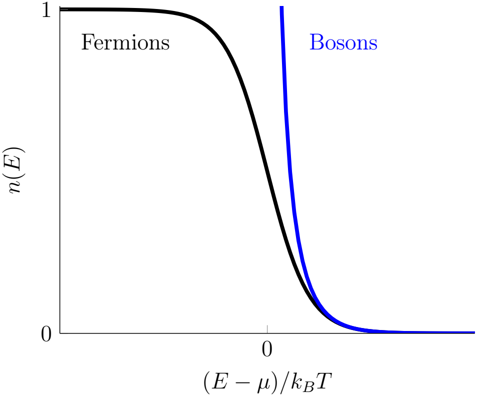
$$n(E) = e^{-(E-\mu)/k_B T}$$

Fermions



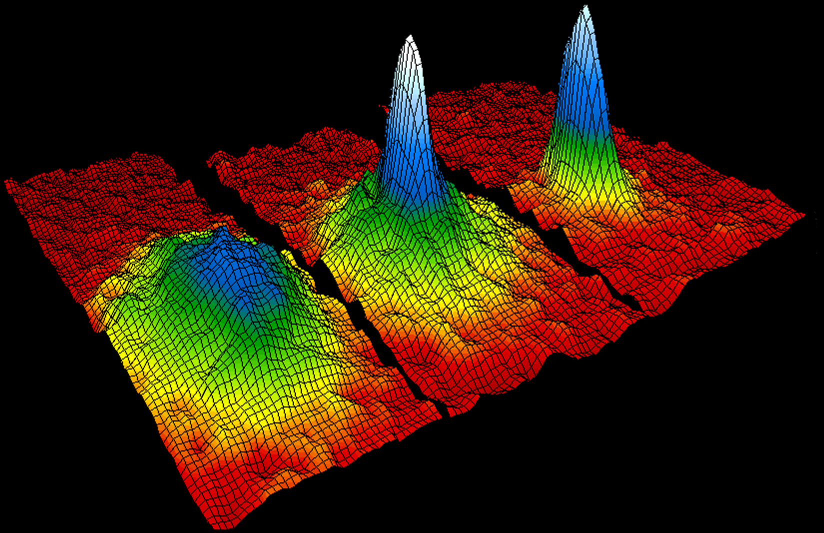
$$n(E) = \frac{1}{e^{(E-\mu)/k_BT}+1}$$

Bosons



$$n(E) = \frac{1}{e^{(E-\mu)/k_BT} - 1}$$

First BEC



Energy

$$n(E) = \frac{1}{e^{(E-\mu)/k_B T} - \eta} \qquad \begin{cases} \eta = 1 & \textit{Boson} \\ \eta = -1 & \textit{Fermion} \end{cases}$$

$$E = \frac{p^2}{2m} + V_{\mathrm{tr}}(\mathbf{x}) + 2gn(\mathbf{x})$$

$$V_{\mathrm{tr}}(\mathbf{x}) = \frac{1}{2} m\omega_\rho^2(\rho^2 + \lambda^2 z^2)$$

$$N = \left(\frac{k_B T}{\hbar\omega}\right)^3 \zeta_3^\eta\left(e^{\mu/k_B T}\right)$$

$$n(\mathbf{x}) = \left(\frac{2\pi\hbar^2}{mk_B T}\right)^{-3/2} \zeta_3^\eta\left(e^{[\mu - V_{\mathrm{tr}}(\mathbf{x}) - 2gn(\mathbf{x})]/k_B T}\right)$$

Dipolar gases

Periodic Table of the Elements

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 1.0079 H Hydrogen																	2 4.0025 He Helium
2	3 6.941 Li Lithium	4 9.0122 Be Beryllium											5 10.811 B Boron	6 12.011 C Carbon	7 14.007 N Nitrogen	8 15.999 O Oxygen	9 18.998 F Flourine	10 20.180 Ne Neon
3	11 22.990 Na Sodium	12 24.305 Mg Magnesium											13 26.982 Al Aluminium	14 28.086 Si Silicon	15 30.974 P Phosphorus	16 32.065 S Sulphur	17 35.453 Cl Chlorine	18 39.948 Ar Argon
4	19 39.098 K Potassium	20 40.078 Ca Calcium	21 44.956 Sc Scandium	22 47.867 Ti Titanium	23 50.942 V Vanadium	24 51.996 **Cr** Chromium	25 54.938 Mn Manganese	26 55.845 Fe Iron	27 58.933 Co Cobalt	28 58.693 Ni Nickel	29 63.546 Cu Copper	30 65.39 Zn Zinc	31 69.723 Ga Gallium	32 72.64 Ge Germanium	33 74.922 As Arsenic	34 78.96 Se Selenium	35 79.904 Br Bromine	36 83.8 Kr Krypton
5	37 85.468 Rb Rubidium	38 87.62 Sr Strontium	39 88.906 Y Yttrium	40 91.224 Zr Zirconium	41 92.906 Nb Niobium	42 95.94 Mo Molybdenum	43 96 Tc Technetium	44 101.07 Ru Ruthenium	45 102.91 Rh Rhodium	46 106.42 Pd Palladium	47 107.87 Ag Silver	48 112.41 Cd Cadmium	49 114.82 In Indium	50 118.71 Sn Tin	51 121.76 Sb Antimony	52 127.6 Te Tellurium	53 126.9 I Iodine	54 131.29 Xe Xenon
6	55 132.91 Cs Caesium	56 137.33 Ba Barium	Lanthanide	72 178.49 Hf Halfnium	73 180.95 Ta Tantalum	74 183.84 W Tungsten	75 186.21 Re Rhenium	76 190.23 Os Osmium	77 192.22 Ir Iridium	78 195.08 Pt Platinum	79 196.97 Au Gold	80 200.59 Hg Mercury	81 204.38 Tl Thallium	82 207.2 Pb Lead	83 208.98 Bi Bismuth	84 209 Po Polonium	85 210 At Astatine	86 222 Rn Radon
7	87 223 Fr Francium	88 226 Ra Radium	Actinide															

57 138.91 La Lanthanum	58 140.12 Ce Cerium	59 140.91 Pr Praseodymium	60 144.24 Nd Neodymium	61 145 Pm Promethium	62 150.36 Sm Samarium	63 151.96 Eu Europium	64 157.25 Gd Gadolinium	65 158.93 Tb Terbium	66 162.50 **Dy** Dysprosium	67 164.93 Ho Holmium	68 167.26 **Er** Erbium	69 168.93 Tm Thulium	70 173.04 Yb Ytterbium	71 174.97 Lu Lutetium
89 227 Ac Actinium	90 232.04 Th Thorium	91 231.04 Pa Protactinium	92 238.03 U Uranium											

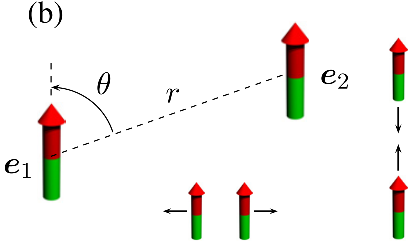
Z mass Symbol Name

Cold dipolar gas milestones

- BEC Cr 2005, 2008
- Dipolar Fermi KRb 2008
- Dipolar Bose KRb 2010
- BEC Dy 2011
- Fermi Dy 2012
- BEC Er 2012

	μ_m/μ_B	$\frac{\text{dipolar}}{\text{contact}} = \frac{g_d}{g}$
$^{87}\mathrm{Rb}$	1	0.0064
$^{52}\mathrm{Cr}$	6	0.15
$^{168}\mathrm{Er}$	7	0.38
$^{164}\mathrm{Dy}$	10	1.3
KRb		20

Dipolar energy



$$E = \frac{p^2}{2m} + V_{\rm tr}(\mathbf{x}) + 2gn(\mathbf{x}) + \Phi_D(\mathbf{x})$$

$$\Phi_D(\mathbf{x}) = \int d\mathbf{x}'\, U_{\rm dd}(\mathbf{x}-\mathbf{x}')n(\mathbf{x}')$$

$$U_{\rm dd}(\mathbf{r}) = \frac{C_{\rm dd}}{4\pi r^3}\left(1 - 3\cos^2\theta\right)$$

$$\tilde{U}_{\rm dd}(\mathbf{k}) = C_{\rm dd}\left(\cos^2\theta_{\mathbf{k}} - 1/3\right)$$

Exchange interaction

Hartree-Fock approximation

$$\langle\hat\psi^\dagger(\mathbf{x})\hat\psi^\dagger(\mathbf{x}')\hat\psi(\mathbf{x}')\hat\psi(\mathbf{x})\rangle \approx \overbrace{n(\mathbf{x})n(\mathbf{x}')}^{\text{Direct/Hartree}} + \overbrace{\eta|\langle\hat\psi^\dagger(\mathbf{x})\hat\psi(\mathbf{x}')\rangle|^2}^{\text{Exchange/Fock}}, \quad \eta = \pm 1$$

Including exchange interaction

$$E = \frac{p^2}{2m} + V_{\mathrm{tr}}(\mathbf{x}) + 2gn(\mathbf{x}) + \Phi_D(\mathbf{x}) + \eta\Phi_E(\mathbf{x},\mathbf{p})$$

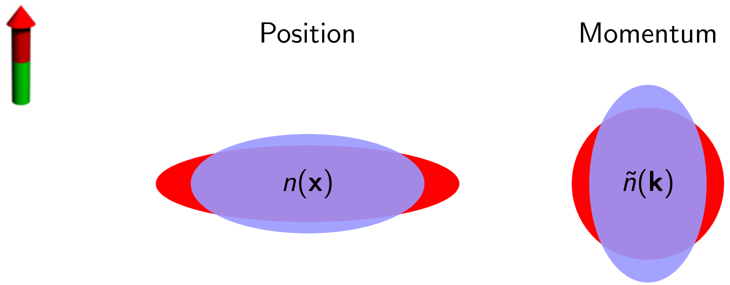
$$W(\mathbf{x},\mathbf{p}) = \frac{1}{e^{(E-\mu)/k_B T} - \eta} \qquad \text{easy}$$

$$\Phi_D(\mathbf{x}) = \int d\mathbf{x}'\, U_{\mathrm{dd}}(\mathbf{x}-\mathbf{x}')n(\mathbf{x}') \qquad \text{harder}$$

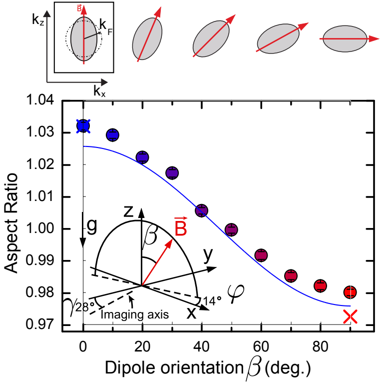
$$\Phi_E(\mathbf{x},\mathbf{p}) = \int d\mathbf{p}'\, \tilde U_{\mathrm{dd}}(\mathbf{p}-\mathbf{p}')W(\mathbf{x},\mathbf{p}') \qquad \text{very hard}$$

Magnetostriction

What is magnetostriction?



Magnetostriction

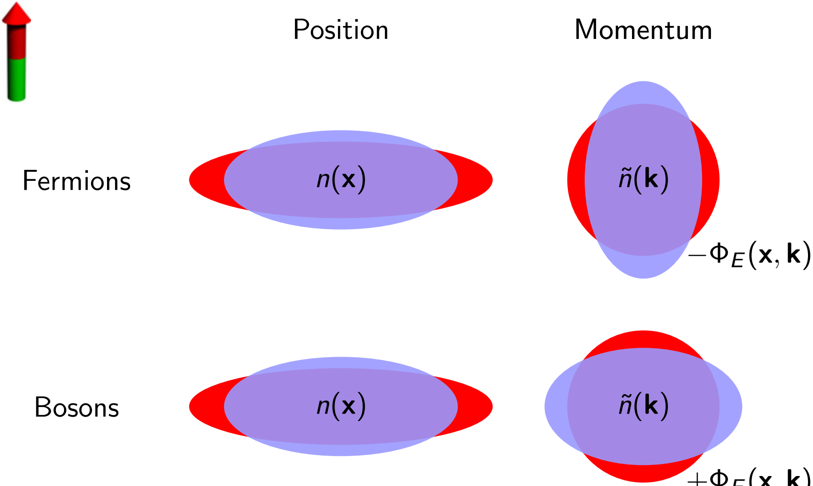


Aikawa, Baier, Frisch, Mark, Ravensbergen, and Ferlaino, "Observation of Fermi surface deformation in a dipolar gas", Science **345**, 1484 (2014)

Baillie and Blakie, "Magnetostriction and exchange effects in trapped dipolar Bose and Fermi gases", Phys. Rev. A **86**, 023605 (2012)

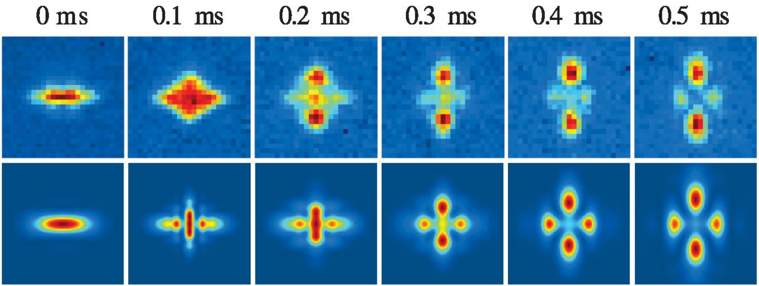
$$\mathrm{AR} = \sqrt{\frac{\langle k_z^2\rangle}{\langle k_\gamma^2\rangle}} = 1 + \left\{1 - \sin^2\beta[1+\cos^2(\gamma-\varphi)]\right\}\frac{3c_\alpha g_d N^{1/6}}{4\pi\hbar\omega a_{\mathrm{ho}}^3},$$

What about bosons?



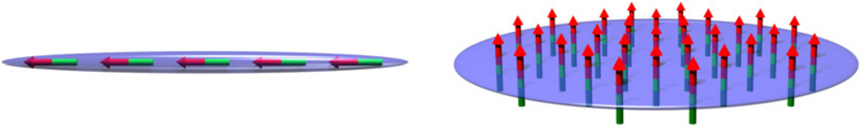
Stability

Dipolar instability



Lahaye, Metz, Fröhlich, Koch, Meister, Griesmaier, Pfau, Saito, Kawaguchi, and Ueda, "*d*-wave collapse and explosion of a dipolar Bose-Einstein condensate", Phys. Rev. Lett. **101**, 080401 (2008)

(a) (b)



Effect of exchange interaction on stability

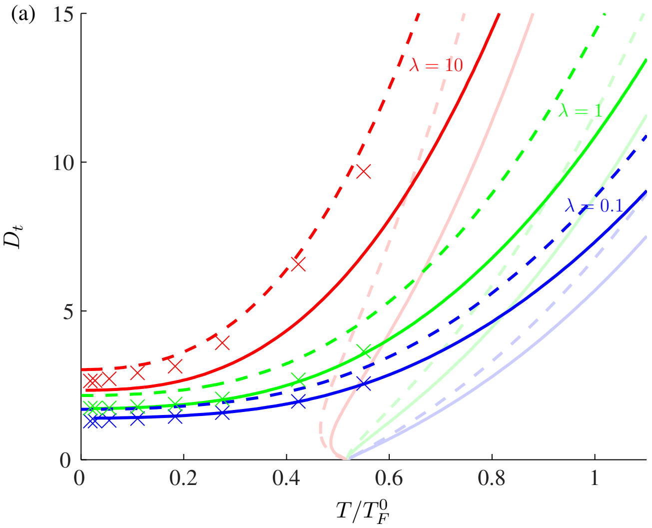
$$\frac{\partial n(\mathbf{x})}{\partial \mu} = \frac{n_\mu(\mathbf{x})}{1 + [2g - C_{\mathrm{dd}}/3 - C_{\mathrm{dd}}\xi_\eta(\mathbf{x})]n_\mu(\mathbf{x})},$$

where we have defined

$$n_\mu(\mathbf{x}) \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} W_\mu(\mathbf{x}, \mathbf{k}),$$

$$\xi_\eta(\mathbf{x}) \equiv -\eta \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{W_\mu(\mathbf{x}, \mathbf{k})}{n_\mu(\mathbf{x})} \frac{\partial \Phi_E(\mathbf{x}, \mathbf{k})}{C_{\mathrm{dd}} \partial n(\mathbf{x})},$$

$$\frac{\partial \Phi_E(\mathbf{x}, \mathbf{k})}{\partial n(\mathbf{x})} = [1 - C_{\mathrm{dd}}\xi_\eta(\mathbf{x})n_\mu(\mathbf{x})] \int \frac{d\mathbf{k}'}{(2\pi)^3} \tilde{U}_{\mathrm{dd}}(\mathbf{k} - \mathbf{k}') \frac{W_\mu(\mathbf{x}, \mathbf{k}')}{n_\mu(\mathbf{x})}$$
$$- \eta \int \frac{d\mathbf{k}'}{(2\pi)^3} \tilde{U}_{\mathrm{dd}}(\mathbf{k} - \mathbf{k}') W_\mu(\mathbf{x}, \mathbf{k}') \frac{\partial \Phi_E(\mathbf{x}, \mathbf{k}')}{\partial n(\mathbf{x})}.$$



Baillie, Bisset, and Blakie, "Stability of a trapped dipolar quantum gas", Phys. Rev. A **91**, 013613 (2015)

Grids

- Position grids Cosine-Hankel (ρ, z): 200×200
- Momentum grids Spherical (p_r, p_θ): 300×30
- Almost 2.9GB to store 4D grid

Matlab on Pan

- Licence
- Parallel options:
  - Built-in for FFT, eigenvalues, matrix multiply, sort
  - Different assumptions as different tasks
  - parfor
  - Distributed Computing Server
  - openMP in mex