

# eResearch 2016

## Dipolar quantum gases

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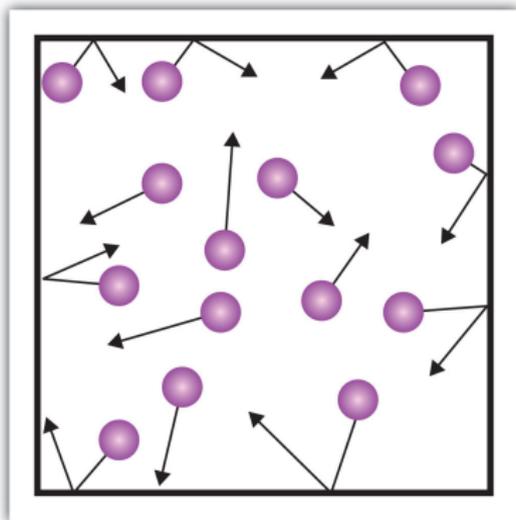


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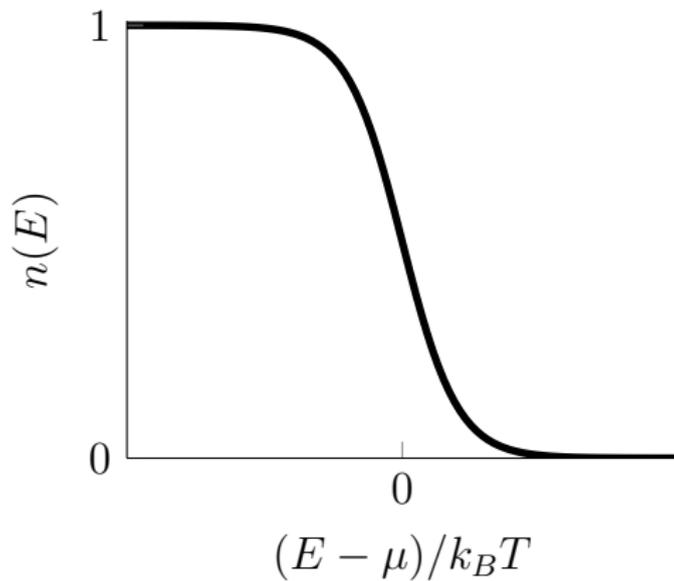


Te Whare Wānanga o Otago  
NEW ZEALAND

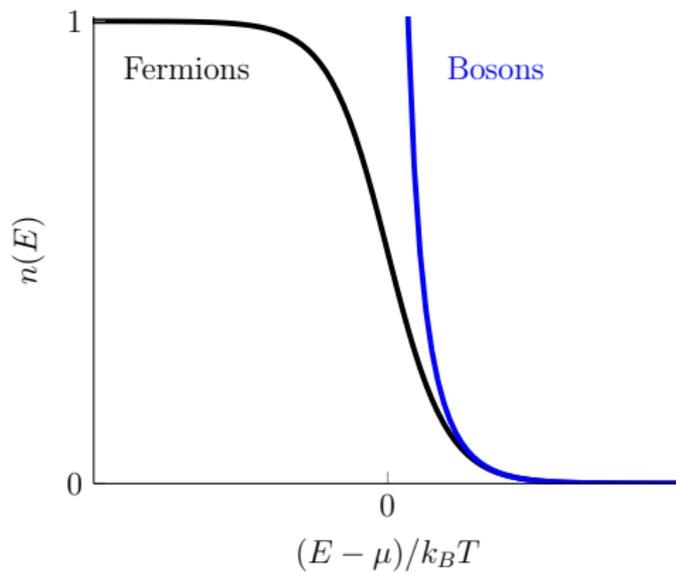
# Gases



$$n(E) = e^{-(E-\mu)/k_B T}$$

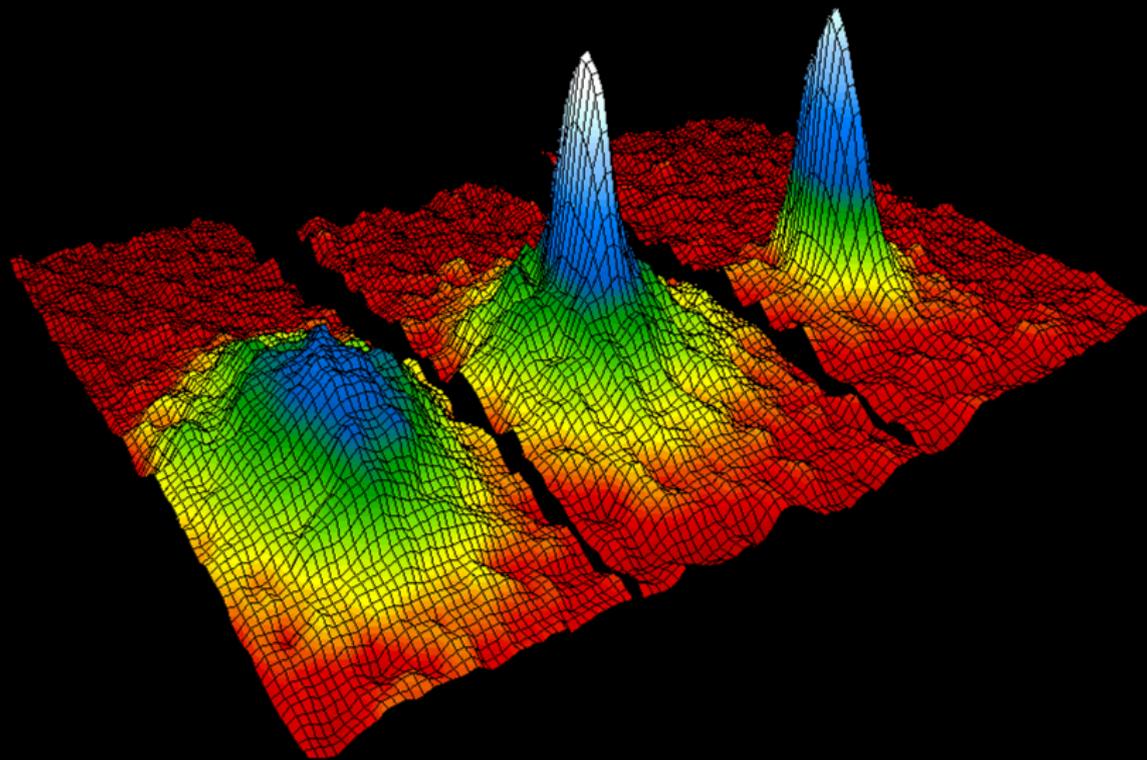


$$n(E) = \frac{1}{e^{(E-\mu)/k_B T} + 1}$$



$$n(E) = \frac{1}{e^{(E-\mu)/k_B T} - 1}$$

# First BEC



$$n(E) = \frac{1}{e^{(E-\mu)/k_B T} - \eta}$$

$$\begin{cases} \eta = 1 & \text{Boson} \\ \eta = -1 & \text{Fermion} \end{cases}$$

$$E = \frac{p^2}{2m} + V_{\text{tr}}(\mathbf{x}) + 2gn(\mathbf{x})$$

$$V_{\text{tr}}(\mathbf{x}) = \frac{1}{2}m\omega_\rho^2(\rho^2 + \lambda^2 z^2)$$

$$N = \left(\frac{k_B T}{\hbar\omega}\right)^3 \zeta_3^\eta(e^{\mu/k_B T})$$

$$n(\mathbf{x}) = \left(\frac{2\pi\hbar^2}{mk_B T}\right)^{-3/2} \zeta_3^\eta(e^{[\mu - V_{\text{tr}}(\mathbf{x}) - 2gn(\mathbf{x})]/k_B T})$$

# Dipolar gases

# Periodic Table of the Elements

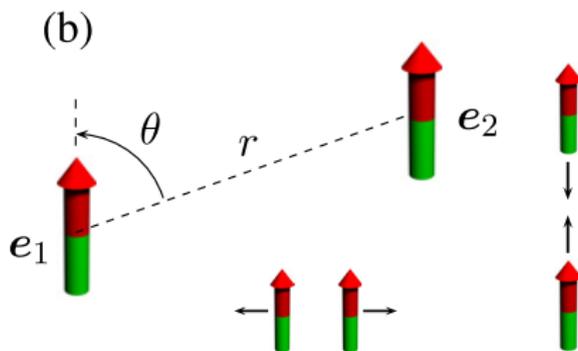
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z mass  
 Symbol  
 Name

# Cold dipolar gas milestones

- BEC Cr 2005, 2008
- Dipolar Fermi KRb 2008
- Dipolar Bose KRb 2010
- BEC Dy 2011
- Fermi Dy 2012
- BEC Er 2012

	$\mu_m/\mu_B$	$\frac{\text{dipolar}}{\text{contact}} = \frac{g_d}{g}$
$^{87}\text{Rb}$	1	0.0064
$^{52}\text{Cr}$	6	0.15
$^{168}\text{Er}$	7	0.38
$^{164}\text{Dy}$	10	1.3
KRb		20



$$E = \frac{p^2}{2m} + V_{\text{tr}}(\mathbf{x}) + 2gn(\mathbf{x}) + \Phi_D(\mathbf{x})$$

$$\Phi_D(\mathbf{x}) = \int d\mathbf{x}' U_{\text{dd}}(\mathbf{x} - \mathbf{x}') n(\mathbf{x}')$$

$$U_{\text{dd}}(\mathbf{r}) = \frac{C_{\text{dd}}}{4\pi r^3} (1 - 3 \cos^2 \theta)$$

$$\tilde{U}_{\text{dd}}(\mathbf{k}) = C_{\text{dd}} (\cos^2 \theta_{\mathbf{k}} - 1/3)$$

# Exchange interaction

## Hartree-Fock approximation

$$\langle \hat{\psi}^\dagger(\mathbf{x}) \hat{\psi}^\dagger(\mathbf{x}') \hat{\psi}(\mathbf{x}') \hat{\psi}(\mathbf{x}) \rangle \approx \underbrace{n(\mathbf{x})n(\mathbf{x}')}_{\text{Direct/Hartree}} + \eta \underbrace{|\langle \hat{\psi}^\dagger(\mathbf{x}) \hat{\psi}(\mathbf{x}') \rangle|^2}_{\text{Exchange/Fock}}, \quad \eta = \pm 1$$

## Including exchange interaction

$$E = \frac{p^2}{2m} + V_{\text{tr}}(\mathbf{x}) + 2gn(\mathbf{x}) + \Phi_D(\mathbf{x}) + \eta\Phi_E(\mathbf{x}, \mathbf{p})$$

$$W(\mathbf{x}, \mathbf{p}) = \frac{1}{e^{(E-\mu)/k_B T} - \eta}$$

$$\Phi_D(\mathbf{x}) = \int d\mathbf{x}' U_{\text{dd}}(\mathbf{x} - \mathbf{x}') n(\mathbf{x}')$$

$$\Phi_E(\mathbf{x}, \mathbf{p}) = \int d\mathbf{p}' \tilde{U}_{\text{dd}}(\mathbf{p} - \mathbf{p}') W(\mathbf{x}, \mathbf{p}')$$

easy

harder

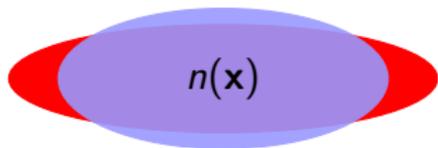
very hard

# Magnetostriction

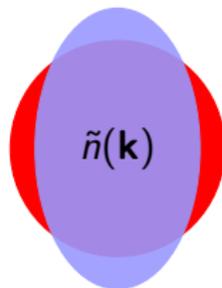
# What is magnetostriction?



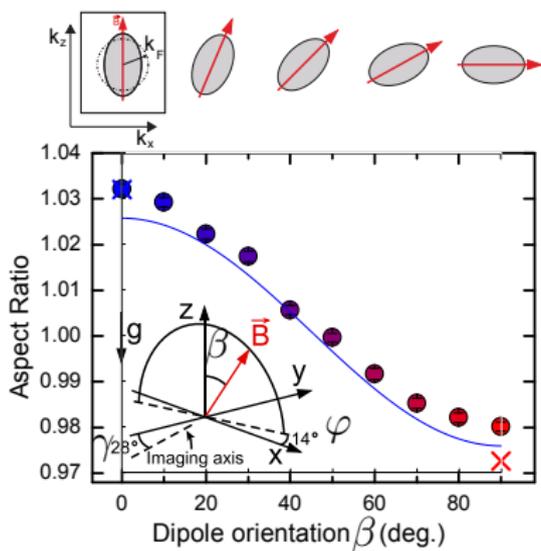
Position



Momentum



# Magnetostriction



Aikawa, Baier, Frisch, Mark, Ravensbergen, and Ferlaino, "Observation of Fermi surface deformation in a dipolar gas", *Science* **345**, 1484 (2014)

Baillie and Blakie, "Magnetostriction and exchange effects in trapped dipolar Bose and Fermi gases", *Phys. Rev. A* **86**, 023605 (2012)

$$AR = \sqrt{\frac{\langle k_z^2 \rangle}{\langle k_y^2 \rangle}} = 1 + \left\{ 1 - \sin^2 \beta [1 + \cos^2(\gamma - \varphi)] \right\} \frac{3c_\alpha g_d N^{1/6}}{4\pi \hbar \omega a_{ho}^3}$$

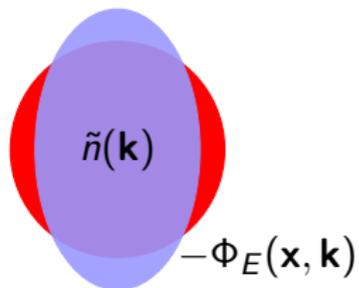
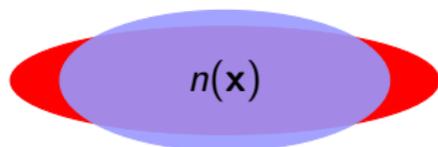
# What about bosons?



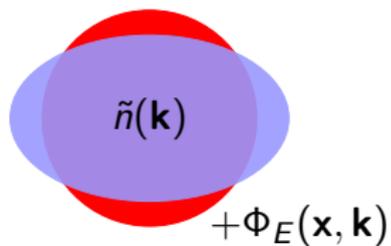
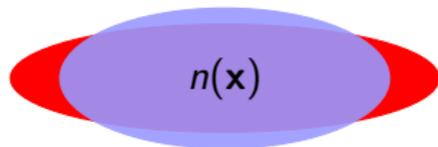
Position

Momentum

Fermions

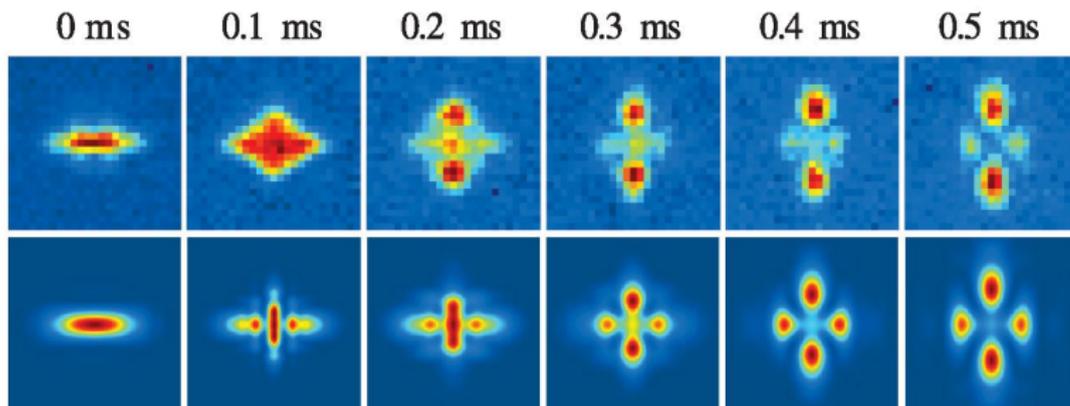


Bosons



# Stability

# Dipolar instability



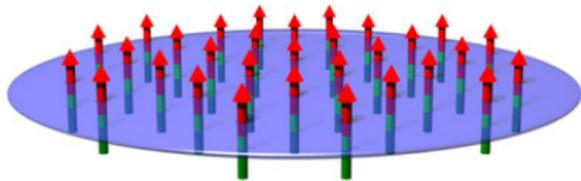
Lahaye, Metz, Fröhlich, Koch, Meister, Griesmaier, Pfau, Saito, Kawaguchi, and Ueda, “*d*-wave collapse and explosion of a dipolar Bose-Einstein condensate”, *Phys. Rev. Lett.* **101**, 080401 (2008)

# Dipolar quantum gases

(a)



(b)



# Effect of exchange interaction on stability

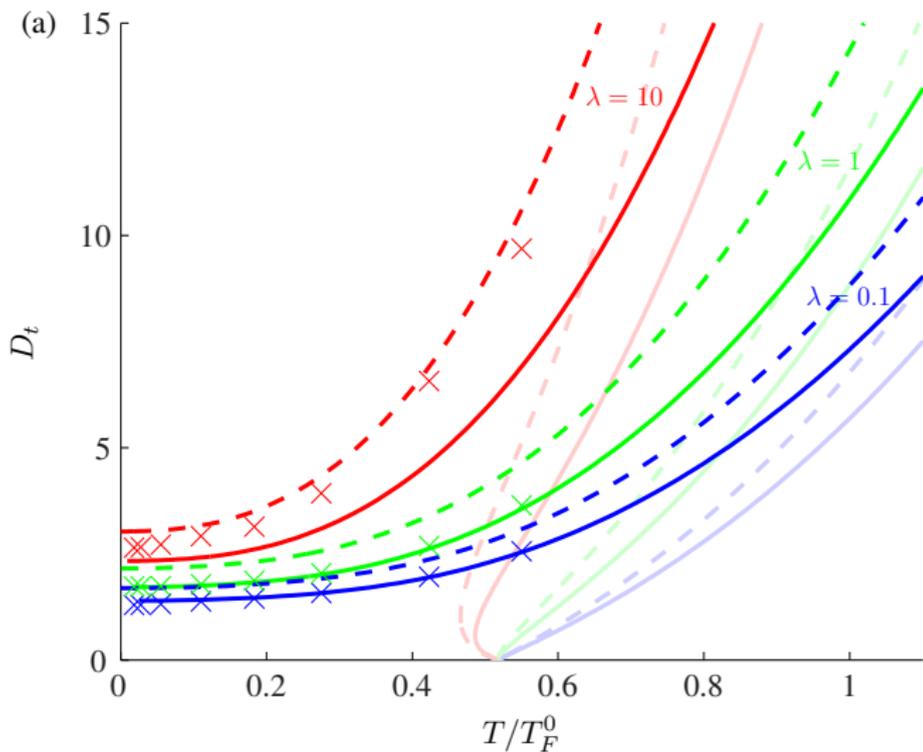
$$\frac{\partial n(\mathbf{x})}{\partial \mu} = \frac{n_\mu(\mathbf{x})}{1 + [2g - C_{\text{dd}}/3 - C_{\text{dd}}\xi_\eta(\mathbf{x})]n_\mu(\mathbf{x})},$$

where we have defined

$$n_\mu(\mathbf{x}) \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} W_\mu(\mathbf{x}, \mathbf{k}),$$

$$\xi_\eta(\mathbf{x}) \equiv -\eta \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{W_\mu(\mathbf{x}, \mathbf{k})}{n_\mu(\mathbf{x})} \frac{\partial \Phi_E(\mathbf{x}, \mathbf{k})}{C_{\text{dd}} \partial n(\mathbf{x})},$$

$$\begin{aligned} \frac{\partial \Phi_E(\mathbf{x}, \mathbf{k})}{\partial n(\mathbf{x})} &= [1 - C_{\text{dd}}\xi_\eta(\mathbf{x})n_\mu(\mathbf{x})] \int \frac{d\mathbf{k}'}{(2\pi)^3} \tilde{U}_{\text{dd}}(\mathbf{k} - \mathbf{k}') \frac{W_\mu(\mathbf{x}, \mathbf{k}')}{n_\mu(\mathbf{x})} \\ &\quad - \eta \int \frac{d\mathbf{k}'}{(2\pi)^3} \tilde{U}_{\text{dd}}(\mathbf{k} - \mathbf{k}') W_\mu(\mathbf{x}, \mathbf{k}') \frac{\partial \Phi_E(\mathbf{x}, \mathbf{k}')}{\partial n(\mathbf{x})}. \end{aligned}$$



Baillie, Bisset, and Blakie, "Stability of a trapped dipolar quantum gas", *Phys. Rev. A* **91**, 013613 (2015)

- Position grids Cosine-Hankel  $(\rho, z)$ :  $200 \times 200$
- Momentum grids Spherical  $(p_r, p_\theta)$ :  $300 \times 30$
- Almost 2.9GB to store 4D grid

- Licence
- Parallel options:
  - Built-in for FFT, eigenvalues, matrix multiply, sort
  - Different assumptions as different tasks
  - parfor
  - Distributed Computing Server
  - openMP in mex